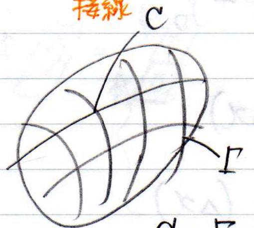
ストークスの定理 ← 数学の定理

$$\oint_{\Gamma上} \underbrace{A\cdot ds}_{接線} = \int_{C上} \underbrace{\mathrm{rot}\,A\cdot dS}_{法線成分}$$

ただし、$A = (A_x, A_y, A_z)$

$$\mathrm{rot}\,A = \begin{cases} \dfrac{\partial A_z}{\partial y} = \dfrac{\partial A_y}{\partial z} \\[2mm] \dfrac{\partial A_x}{\partial z} = \dfrac{\partial A_z}{\partial x} \\[2mm] \dfrac{\partial A_y}{\partial x} = \dfrac{\partial A_x}{\partial y} \end{cases}$$

rotation：（回転）



C：Γで張られる曲面

$$\therefore \underbrace{\oint_{\Gamma上} E\cdot ds}_{=0} \overset{\text{Stokes}}{=} \int_{C上} \mathrm{rot}\,E\cdot dS$$

Γ, Cは任意

$$\therefore \boxed{\mathrm{rot}\,E = 0} \to 微分形$$

エネルギー保存則

積分形	$\oint_{\Gamma上} E\cdot ds = 0$
微分形	$\mathrm{rot}\,E = 0$

→ クーロン場は いたるところで渦なし.

$$\begin{cases} \mathrm{div}\,E = \dfrac{1}{\varepsilon_0}\rho(r) \\ \mathrm{rot}\,E = 0 \end{cases}$$

→ 境界条件を与えれば、$E(r)$ は一意に決まる.

- rotation の意味

ストークスの定理で、任意の閉曲線を、ある点(x, y, z)の
まわりの微小な線とすると、

$$\oint_{\Gamma上} A\cdot ds = \int_{C上} \mathrm{rot}\,A\cdot dS \simeq (\mathrm{rot}\,A)_n \times a$$

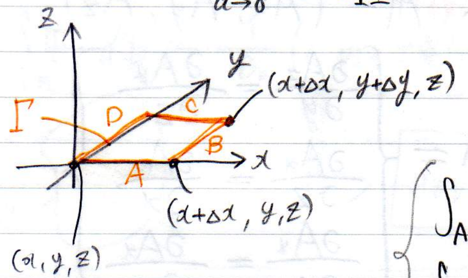
normal　　a(面積)　Γ

$$\therefore \boxed{(\mathrm{rot}\,A)_n = \lim_{a\to 0}\frac{1}{a}\oint_{\Gamma上} A\cdot ds} \Leftrightarrow (\mathrm{rot}\,A)_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}$$

ある点のまわりの循環の単位面積あたりの値が法線成分.

$$\mathrm{rot}\,A = 0 \Leftrightarrow 渦なし.$$

$$(\mathrm{rot}\,A)_n = \lim_{a\to 0}\frac{1}{a}\oint_{\Gamma上} A\cdot ds \iff \text{例えば、}\ (\mathrm{rot}\,A)_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}$$



$$\oint_{\Gamma上} A\cdot ds = \int_A + \int_B + \int_C + \int_D$$

$$\left\{\begin{aligned} \int_{A上} &= A_x\left(x+\frac{\Delta x}{2}, y, z\right)\cdot(\Delta x) \\ \int_{C上} &= -A_x\left(x+\frac{\Delta x}{2}, y+\Delta y, z\right)\cdot(\Delta x) \end{aligned}\right\}$$

- $$\int_{A上} + \int_{C上} = (\Delta x)\cdot\left[A_x\left(x+\frac{\Delta x}{2}, y, z\right) - A_x\left(x+\frac{\Delta x}{2}, y+\Delta y, z\right)\right]$$

$$= -(\Delta x)(\Delta y)\frac{\partial}{\partial y}A_x\left(x+\frac{\Delta x}{2}, y, z\right)$$

$$\left\{\begin{aligned} \int_{B上} &= A_y\left(x+\Delta x, y+\frac{\Delta y}{2}, z\right)\cdot(\Delta y) \\ \int_{D上} &= -A_y\left(x, y+\frac{\Delta y}{2}, z\right)\cdot(\Delta y) \end{aligned}\right\}$$

- $$\int_{B上} + \int_{D上} = (\Delta y)(\Delta x)\frac{\partial}{\partial x}A_y\left(x, y+\frac{\Delta y}{2}, z\right)$$

$$\oint_{\Gamma上} = \int_A + \int_B + \int_C + \int_D$$

$$= \underbrace{(\Delta x)(\Delta y)}_{= a}\left[-\frac{\partial}{\partial y}A_x\left(x+\frac{\Delta x}{2}, y, z\right) + \frac{\partial}{\partial x}A_y\left(x, y+\frac{\Delta y}{2}, z\right)\right]$$

$$\frac{1}{a}\oint_{\Gamma上} A\cdot ds = \frac{\partial A_y}{\partial x}\left(x, y+\frac{\Delta y}{2}, z\right) - \frac{\partial A_x}{\partial y}\left(x+\frac{\Delta x}{2}, y, z\right)$$

$$\lim_{a\to 0}\frac{1}{a}\oint_{\Gamma上} A\cdot ds = \frac{\partial A_y}{\partial x}(x, y, z) - \frac{\partial A_x}{\partial y}(x, y, z) = (\mathrm{rot}\,A)_z$$