$$\therefore E_x dx + E_y dy + E_z dz = -\left(\frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz\right)$$

dx, dy, dz は任意

$$\begin{cases} E_x = -\dfrac{\partial \phi}{\partial x} \\ E_y = -\dfrac{\partial \phi}{\partial y} \\ E_z = -\dfrac{\partial \phi}{\partial z} \end{cases} \qquad \boxed{\mathbb{E} = -\operatorname{grad}\phi}$$

(定義) $\begin{cases} (\operatorname{grad} f)_x = \dfrac{\partial f}{\partial x} \\ (\operatorname{grad} f)_y = \dfrac{\partial f}{\partial y} \\ (\operatorname{grad} f)_z = \dfrac{\partial f}{\partial z} \end{cases}$: $f(x,y,z)$ の勾配 (gradient)　関数

{ 向き：最も傾きが急な方向
　大きさ：その方向の勾配 }

$\operatorname{rot}\mathbb{E} = 0 \longrightarrow \phi(x,y,z)$ が存在 $\longrightarrow \mathbb{E} = -\operatorname{grad}\phi(x,y,z)$

スカラ関数 ϕ ：静電ポテンシャル

$\because \operatorname{rot}(\operatorname{grad}\phi) \equiv 0$ 恒等的に。

$$\boxed{\mathbb{E}\text{の単位：} \; N/C = V/m \qquad (\mathbb{F} = q\mathbb{E}, \quad \mathbb{E} = -\operatorname{grad}\phi)}$$

$$\begin{cases} \operatorname{div}\mathbb{E} = \dfrac{1}{\varepsilon_0}\rho(\mathbf{r}) \\ \mathbb{E} = -\operatorname{grad}\phi \end{cases}$$ 代入。

$$\operatorname{div}\mathbb{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right)$$
$$= -\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right]$$
$$= -\Delta\phi$$

$$\Delta \equiv \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$ ：ラプラシアン（ラプラス演算子）

$\rho(\boldsymbol{r})$が与えられたとき、境界条件を与えると、$\phi(\boldsymbol{r})$は一意に定まる

$$\boxed{\Delta\phi = -\frac{1}{\varepsilon_0}\rho(\boldsymbol{r})}$$ → Poisson 方程式

特に $\rho=0$ のとき、 $\boxed{\Delta\phi = 0}$ Laplace 方程式

クーロンの法則

$$\mathbb{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\boldsymbol{r}-\boldsymbol{r}'|^3}(\boldsymbol{r}-\boldsymbol{r}') \Longleftrightarrow \begin{cases} \mathrm{div}\,\mathbb{E} = \frac{1}{\varepsilon_0}\rho(\boldsymbol{r}) \\ \mathrm{rot}\,\mathbb{E} = 0 \end{cases}$$

Gaussの法則

エネルギー保存則

$$\Longleftrightarrow \begin{cases} \Delta\phi = -\frac{1}{\varepsilon_0}\rho(\boldsymbol{r}) \\ \mathbb{E} = -\mathrm{grad}\,\phi(\boldsymbol{r}) \end{cases}$$

(cf) 微分演算子について、

ナブラ

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\nabla\cdot\nabla = \Delta$$ ラプラシアン

$$\left(\begin{array}{ll} \mathrm{grad}\,f = \nabla f & (\text{スカラー} \to \text{ベクトル}) \\ \mathrm{div}\,\mathbb{A} = \nabla\cdot\mathbb{A} & (\text{ベクトル} \to \text{スカラー}) \\ \mathrm{rot}\,\mathbb{A} = \nabla\times\mathbb{A} & (\text{ベクトル} \to \text{ベクトル}) \\ \Delta f = \nabla^2 f & (\text{スカラー} \to \text{スカラー}) \end{array}\right)$$

(cf) Laplacian

$$\Delta\phi = 0$$ ：Laplace eq.

$$\Delta\phi = \frac{1}{v^2}\cdot\frac{\partial^2}{\partial t^2}\phi$$ ：波動方程式

$$\Delta\phi = D^{-1}\frac{\partial}{\partial t}\phi$$ ：拡散方程式

$$\left[-\frac{\hbar^2}{2m}\Delta + V(\boldsymbol{r})\right]\phi = i\hbar\frac{\partial}{\partial t}\phi$$ ：Schrödinger eq.