

H.21 物理学基礎理論B.

(1) $A = (-y, x, -z)$ の回転と発散

(i) 発散に ついて

$$\begin{aligned} \operatorname{div} A &= \nabla \cdot A = \\ &= \frac{\partial}{\partial x^1} A_1(x^1, x^2, x^3) + \frac{\partial}{\partial x^2} A_2(x^1, x^2, x^3) \\ &\quad + \frac{\partial}{\partial x^3} A_3(x^1, x^2, x^3) \end{aligned}$$

∴ $\nabla \cdot A = 2$

$$\begin{aligned} \nabla \cdot A &= 0 + 0 + (-1) \\ &= -1 \quad \# \end{aligned}$$

(ii) 回転に ついて

$$\begin{aligned} \operatorname{curl} A &= \nabla \times A \\ &= \left(\left(\frac{\partial A_3}{\partial x^2} - \frac{\partial A_2}{\partial x^3} \right) e_1, \left(\frac{\partial A_1}{\partial x^3} - \frac{\partial A_3}{\partial x^1} \right) e_2, \left(\frac{\partial A_2}{\partial x^1} - \frac{\partial A_1}{\partial x^2} \right) e_3 \right) \end{aligned}$$

である。

$(-y, x, -z)$ の場合、

$$\begin{array}{cccc} \textcircled{e_1} & \textcircled{e_2} & \textcircled{e_3} & e_i \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \\ -y & x & -z & -y \end{array}$$

$$\begin{aligned} \nabla \times A &= \left((0 - 0), (0 - 0), (1 - (-1)) \right) \end{aligned}$$

$$= (0, 0, 2) \quad \#$$