No. ・ ・ (　　)

H.22 物理学基礎論

(1) $f(x) = (1-3x+2x^2)^{-\frac{1}{2}}$

$$\frac{df(x)}{dx} = -\frac{1}{2}\cdot((1-3x+2x^2)^{-\frac{3}{2}}\cdot(-3+4x)$$

$$\frac{d^2f(x)}{dx^2} = -\frac{1}{2}\cdot\left\{-\frac{3}{2}(1-3x+2x^2)^{-\frac{5}{2}}(-3+4x)^2 + 4\cdot(1-3x+2x^2)^{-\frac{3}{2}}\right\}$$

$-\frac{1}{2}$ $-\frac{27}{2}$ $-\frac{3}{2}\cdot 9$ $+4$ $\frac{5}{2}$

$$\therefore f(x) = f(0) + \left.\frac{df(x)}{dx}\right|_{x=0} + \left.\frac{d^2f(x)}{dx^2}\right|_{x=0}$$

$$= 1 - \frac{1}{2}\cdot 1\cdot(-3)\cdot x + \frac{1}{2!}\cdot\left(-\frac{1}{2}\right)\cdot\left(-\frac{19}{2}\right)x^2$$

$$= \underline{1 + \frac{3}{2}x + \frac{19}{8}x^2}$$

(2) $x(0) = 1,\ \dot{x}(0) = -\frac{1}{2}$.

$$\frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} + w^2x(t) = 0$$

$x(0) = 1,$

$\dot{x}(0) = -\frac{1}{2},$

$$\left.\frac{d^2x(t)}{dt^2}\right|_{t=0} = -\left.\frac{dx(t)}{dt}\right|_{t=0} - w^2\left.x(t)\right|_{t=0}$$

$$= \underline{\frac{1}{2} - w^2}$$

$$\left.\frac{d^3x(t)}{dt^3}\right|_{t=0} = -\left.\frac{d^2x(t)}{dt^2}\right|_{t=0} - w^2\left.\frac{dx(t)}{dt}\right|_{t=0}$$

$$= -\left(\frac{1}{2}-w^2\right) - w^2\cdot\left(-\frac{1}{2}\right)$$

$$= -\frac{1}{2} + w^2 + \frac{1}{2}w^2$$

$$= \frac{3}{2}w^2 - \frac{1}{2}$$

$$\left.\frac{d^4x(t)}{dt^4}\right|_{t=0} = -\left.\frac{d^3x(t)}{dt^3}\right|_{t=0} - w^2\left.\frac{d^2x(t)}{dt^2}\right|_{t=0}$$

$$= -\left(\frac{3}{2}w^2 - \frac{1}{2}\right) - w^2\times\left(\frac{1}{2}-w^2\right)$$

$$= w^4 - 2w^2 + \frac{1}{2}$$

$\vdots$

$$\therefore x(t) = 1 - \frac{1}{2}x + \frac{1}{2!}\left(\frac{1}{2}-w^2\right)x^2 + \frac{1}{3!}\left(\frac{3}{2}w^2-\frac{1}{2}\right)x^3 + \frac{1}{4!}\left(w^4-2w^2+\frac{1}{2}\right)x^4 + \cdots$$

$$= \left\{1 + \frac{1}{2!}\left(\frac{1}{2}-w^2\right)x^2 + \frac{1}{4!}\left(w^4-2w^2+\frac{1}{2}\right)x^4 + \cdots\right\} + \left\{-\frac{1}{2}x + \frac{1}{3!}\left(\frac{3}{2}w^2-\frac{1}{2}\right)x^3 + \cdots\right\}$$