平成 18 年度 T-3 計算と論理

設問 1

1.

P	Q	$P \supset Q$	$(P \supset Q) \vee Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

2.

P	Q	$Q \supset P$	$P \supset (Q \supset P)$	$(P \supset (Q \supset P)) \supset P$
T	T	T	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

3.

P	Q	R	$P \supset Q$	$(P \supset Q) \wedge R$	$R \supset Q$	$((P \supset Q) \wedge R) \supset (R \supset Q)$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	F	T	T

4.

P	Q	R	$P \wedge Q$	$(P \wedge Q) \vee R$	$P \vee R$	$Q \vee R$	$(P \vee R) \wedge (Q \vee R)$	$((P \wedge Q) \vee R) \supset ((P \vee R) \wedge (Q \vee R))$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	T	F	F	T
F	T	T	F	T	T	T	T	T
F	T	F	F	F	F	T	F	T
F	F	T	F	T	T	T	T	T
F	F	F	F	F	F	F	F	T

設問 2

(1)

$$
\dfrac{\dfrac{\dfrac{\dfrac{\overline{\{(\exists x.A(x)) \supset B\} \cup \{A(x)\} \vdash (\exists x.A(x)) \supset B} \qquad \dfrac{\overline{\{(\exists x.A(x)) \supset B\} \cup \{A(x)\} \vdash A(x)}}{\{(\exists x.A(x)) \supset B\} \cup \{A(x)\} \vdash \exists x.A(x)}}{\{(\exists x.A(x)) \supset B\} \cup \{A(x)\} \vdash B}}{\{(\exists x.A(x)) \supset B\} \vdash A(x) \supset B}}{\{(\exists x.A(x)) \supset B\} \vdash \forall x.(A(x) \supset B)}}{\vdash ((\exists x.A(x)) \supset B) \supset \forall x.(A(x) \supset B)}
$$

(2)

$$
\dfrac{\dfrac{\dfrac{\overline{\Gamma \cup \{\exists x.A(x)\} \vdash \exists x.A(x)} \qquad \dfrac{\dfrac{\dfrac{\overline{\Gamma \cup \{\exists x.A(x)\} \cup \{A(y)\} \vdash \forall x.(A(x) \supset B(x))}}{\Gamma \cup \{\exists x.A(x)\} \cup \{A(y)\} \vdash A(y) \supset B(y)} \qquad \overline{\Gamma \cup \{\exists x.A(x)\} \cup \{A(y)\} \vdash A(y)}}{\Gamma \cup \{\exists x.A(x)\} \cup \{A(y)\} \vdash B(y)}}{\Gamma \cup \{\exists x.A(x)\} \cup \{A(y)\} \vdash \exists y.B(y)}}{\Gamma \cup \{\exists x.A(x)\} \vdash \exists y.B(y)}}{\Gamma \vdash (\exists x.A(x)) \supset \exists y.B(y)}}{\vdash (\forall x.(A(x) \supset B(x))) \supset ((\exists x.A(x)) \supset \exists y.B(y))}
$$

$\Gamma = \{\forall x.(A(x) \supset B(x))\}$

設問 3

1.

$$\dfrac{\dfrac{}{\texttt{(+ (s (s z)) z)} \to \texttt{(s (+ (s z) z))}}}{\texttt{(+ (+ (s (s z)) z) (s z))} \to \texttt{(+ (s (+ (s z) z)) (s z))}}$$

$$\dfrac{\dfrac{\dfrac{}{\texttt{(+ (s z) z)} \to \texttt{(s (+ z z))}}}{\texttt{(s (+ (s z) z))} \to \texttt{(s (s (+ z z)))}}}{\texttt{(+ (s (+ (s z) z)) (s z))} \to \texttt{(+ (s (s (+ z z))) (s z))}}$$

$$\dfrac{}{\texttt{(+ (s (+ (s z) z)) (s z))} \to \texttt{(s (+ (+ (s z) z) (s z)))}}$$

$$\begin{aligned}[e_1,\ e_2] \quad &= \quad [\texttt{(+ (s (+ (s z) z)) (s z))},\ \texttt{(+ (s (s (+ z z))) (s z))}] \\ , \quad & \quad [\texttt{(+ (s (+ (s z) z)) (s z))},\ \texttt{(s (+ (+ (s z) z) (s z)))}]\end{aligned}$$

2.

(i) $n_1 = \texttt{z}$ のとき

$$\dfrac{\dfrac{}{\texttt{(+ z } n_2\texttt{)} \to n_2}}{\texttt{(+ z } n_2\texttt{)} \to^{+} n_2}$$

$n_3 = n_2$ と取れば $n_3 \in N$ となり成立する。

(ii) $n_1 = \texttt{(s } n'\texttt{)}$ のとき

$n_1 = n'$ のときの成立を仮定することにより

$$\dfrac{\dfrac{}{\texttt{(+ (s } n'\texttt{) } n_2\texttt{)} \to^{+} \texttt{(s (+ } n'\ n_2\texttt{))}} \qquad \dfrac{\texttt{(+ } n'\ n_2\texttt{)} \to^{+} n_3'}{\texttt{(s (+ } n'\ n_2\texttt{))} \to^{+} \texttt{(s } n_3'\texttt{)}}}{\texttt{(+ (s } n'\texttt{) } n_2\texttt{)} \to^{+} \texttt{(s } n_3'\texttt{)}}$$

$n_3 = \texttt{(s } n_3'\texttt{)}$ と取れば $n_3 \in N$ となり成立する。

(i), (ii) により、題意は帰納的に証明された。